[5]Nowacki, W., "Some General Theorems of Thermopiezoelectricity," *Journal of Thermal Stresses*, Vol. 1, No. 2, 1978, pp. 171–182.
[6]Maciejowski, J. M., *Multivariable Feedback Design*, Addison–Wesley, Reading, MA, 1989, pp. 223, 224.

Lateral Directional Aircraft Control Using Eigenstructure Assignment

William L. Garrard*
University of Minnesota, Minneapolis, Minnesota 55455

Introduction

EIGENSTRUCTURE assignment has received considerable attention as a method for design of flight control systems.[1–3] Although recently overshadowed by various H_∞-based techniques in the research literature, eigenstructure techniques offer a simple method for designing low-order, easily implementable controllers that can directly incorporate handling quality specifications. It is well known that, for a controllable linear system, eigenstructure techniques can be used to design a linear feedback controller that places closed-loop eigenvalues and shapes closed-loop eigenvectors into desired configurations.[4] The question for the designer is how to specify these desired eigenvalues and eigenvectors. The approach developed in this Note is to use a state-space model of desired responses that is based on flying quality specifications. This state-space model is then used to generate the desired eigenvalues and eigenvectors, which are to be obtained by feedback control. In this Note, a direct method is used for calculating the gain matrix, which achieves the desired eigenvalues exactly and the desired eigenvectors in a best least squares sense. Direct methods for eigenstructure assignment by least squares methods are described in numerous references.[1–6] Garrard et al.[5] and Low and Garrard[6] showed how to use handling quality specifications for helicopters to generate desired eigenstructures, which were then achieved in a least squares sense by feedback control. In this Note, the technique is extended to lateral-directional control of an aircraft. As an illustrative example, control of a tailless aircraft is considered. The method of state-space modeling of handling qualities could also be used in design of model matching H_2 or H_∞ controllers.

In a recent paper Mengali[7] developed a somewhat different approach to eigenstructure design in which he fixes a priori the components of the desired eigenvectors and then numerically minimizes a linear quadratic performance index to achieve the desired eigenvalues within specified limits. There are two major differences between the control design methodology in this Note and that of Mengali's. These are 1) the way in which the desired eigenstructure is selected and 2) the way in which the gain matrix is calculated. Mengali considers the control of a large transport. The open-loop response of this aircraft is relatively well behaved compared to the very unstable aircraft considered in this Note. It is not clear that Mengali's approach to closed-loop eigenvector selection would work well in the case of an extremely unstable aircraft. On the other hand, the method for eigenvector selection described in this Note could be used with Mengali's method for gain matrix calculation. This is an interesting idea because the direct method may, in some cases, result in large actuator deflections, whereas Mengali's method automatically trades off eigenvalue location (bandwidth) with surface activity. This tradeoff would have to be done by hand using the direct method.

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*Professor and Head, Department of Aerospace Engineering and Mechanics. E-mail: wgarrard@aem.umn.edu. Associate Fellow AIAA.

Design Technique

The system to be controlled is given in linear state-space form as

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} \tag{1}$$

and the control is a linear function of the state vector

$$\boldsymbol{u} = -K\boldsymbol{x} \tag{2}$$

The feedback gain matrix is selected such that this control law results in desired placement of closed-loop eigenvalues and shaping of closed-loop eigenvectors. In the method presented, the desired eigenstructure is derived from a state-space model that represents the desired response. This model is given by

$$\dot{\boldsymbol{x}}_d = A_d\boldsymbol{x}_d + B_d\boldsymbol{x}_c \tag{3}$$

The question is how to select A_d and B_d to give specified handling qualities. It is widely accepted that a first-order, roll-rate response and a second-order, side-slip response are desirable for acceptable handling qualities. Although many papers have developed controllers that decouple roll rate and side slip, it is clear from a knowledge of basic handling qualities that this is not desirable for stability augmentation because it implies neutral static lateral stability, i.e., L_β. (Neutral lateral stability may be desirable in a control augmentation system.) Also a stable, long time constant, spiral response is beneficial to good handling qualities.[8]

Desired lateral-directional response is modeled as

$$\begin{bmatrix}\dot{\beta}\\ \dot{\phi}\\ \dot{p}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}0 & \dfrac{g\cos\theta}{V} & 0 & -1\\ 0 & 0 & 1 & 0\\ L_\beta & 0 & L_p & 0\\ N_\beta & 0 & 0 & N_r\end{bmatrix}\begin{bmatrix}\beta\\ \phi\\ p\\ r\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ -L_p & 0\\ 0 & -N_\beta\end{bmatrix}\begin{bmatrix}p_c\\ \beta_c\end{bmatrix} \tag{4}$$

There is no cross coupling in the B_d matrix to avoid a roll-rate command as input to the yaw axis or vice versa. Because A_d determines the eigenstructure, it is specified so that it has a first-order roll subsidence mode, a second-order Dutch roll mode, a stable spiral mode, and static lateral stability. The characteristic equation for A_d is

$$(L_p - s)\left(-s^3 + N_r s^2 - N_\beta s\right) + \left(\frac{g\cos\theta}{V}\right)(N_r - s)L_\beta = 0 \tag{5}$$

If $g\cos\theta/V$ is small, then the nonzero roots of this equation are

$$s = L_p, \qquad s = (N_r/2) \pm \tfrac{1}{2}\sqrt{N_r - 4N_\beta} \tag{6}$$

Assuming s small to get the spiral root yields

$$\left(L_pN_\beta + L_\beta\frac{g\cos\theta}{V}\right)s - \left(\frac{g\cos\theta}{V}\right)N_rL_\beta = 0 \tag{7}$$

Substituting numerical values in these approximations gives

$$s = -4, \qquad s = -0.0389, \qquad s = -0.5 \pm j1.9365$$

The actual eigenvalues of A_d are

$$s = -4.0289, \qquad s = -0.0382, \qquad s = -0.4665 \pm j1.9581$$

The approximations just given are very similar to the three-degree-of-freedom Dutch roll approximation of McRuer et al.[9] for Y_β zero and if the derivatives are prime. It is easy to include Y_β in the formulation just given; however, in the flying quality specifications, no unambiguous specification of the desired value of this variable appears to exist and so it was set equal to zero for the sake of simplicity.

Except for very low velocities, the $g\cos\theta/V$ term is small but provides a stable spiral response. The L_p term is the negative inverse of the roll-subsidence time constant T_R. N_β is approximately the square of the Dutch roll frequency, and N_r supplies the Dutch roll damping. The values for these terms can be selected from handling quality specifications. These specifications also provide L_β as a function of T_R, so that once the time constant for the roll-subsidence is selected, L_β can be determined.[9]

Once values have been assigned to the terms in the A_d matrix, it can be used to generate a desired set of eigenvalues and eigenvectors for eigenstructure controller design or transfer function models for H_2- or H_∞-based model matching design approaches.

Example

To illustrate the technique, the results are applied to the control of a mathematical model of a tailless fighter aircraft.[10] The model considered is a hypothetical F-16 with its vertical tail surface area reduced to 20% of its nominal value and thrust vectoring added to replace the loss of the rudder. The linearized open-loop dynamics of the aircraft are given in the form of Eq. (1) at $M = 0.56$ and sea level:

$$A = \begin{bmatrix} -0.1024 & 0.0519 & 0.0155 & 0.9996 \\ 0 & 0 & 1.0000 & 0.0155 \\ -14.1271 & 0 & -4.5338 & 0.0633 \\ -10.5446 & 0 & -0.0241 & -0.0377 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0209 & 0.0075 \\ 0 & 0 \\ -63.7748 & -0.0930 \\ -2.8262 & -0.8995 \end{bmatrix}$$

The control vector comprises the aileron deflection and the angular deflection of the thrust vector nozzle in the yaw direction.

The open-loop eigenvalues are 3.1398, −3.2737, 0.002, and −4.5422. The pole at −4.5422 is associated with roll subsidence, the small positive real root is associated with an unstable spiral mode, and the real eigenvalues at 3.1398 and −3.2737 are associated with eigenvectors that contain both rolling and yawing rates and could be considered as Dutch roll modes. In any case, the large positive eigenvalue would result in response that is so unstable that the open-loop aircraft would be unflyable.

To put the model in the form given by Eq. (3), it is necessary to let

$$\boldsymbol{u} = H\boldsymbol{x}_c \tag{8}$$

where

$$B \times H \approx B_d \tag{9}$$

H can be determined using the pseudoinverse method.

If we select $\omega_{\mathrm{DR}} = 2$ rad/s, $\zeta_{\mathrm{DR}} = 0.25$, and $T_R = 0.25$ s, then from the handling quality specifications,[8] $L_\beta = -12$. This results in

$$A_d = \begin{bmatrix} 0 & 0.0519 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ -12 & 0 & -4 & 0 \\ 4 & 0 & 0 & -1 \end{bmatrix}$$

and

$$B_d = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 4 & 0 \\ 0 & -4 \end{bmatrix}$$

Using the pseudoinverse,

$$H = \begin{bmatrix} -0.0630 & -0.0065 \\ 0.1980 & 4.4671 \end{bmatrix}$$

and

$$B \times H = \begin{bmatrix} 0.0002 & 0.0334 \\ 0.0000 & 0.0000 \\ 3.9994 & -0.0009 \\ -0.0001 & -3.9998 \end{bmatrix}$$

The eigenvalues of A_d are as follows: Dutch roll $-0.5267 \pm 1.9722i$ ($\omega = 2.0413$, $\zeta = 0.2580$), roll subsidence −4.0296, and spiral −0.0371, and the respective eigenvectors are

$$\begin{matrix} 0.2408 \pm 0.0649i, & -0.0024, & -0.0122 \\ 0.3166 \pm 0.1856i, & 0.2409, & -0.9979 \\ -0.5328 \pm 0.5266i, & -0.9706, & 0.0370 \\ -0.0136 \pm 0.4916i, & 0.0032, & -0.0508 \end{matrix}$$

These values make the unstable tailless aircraft respond like a conventional stable aircraft that exhibits reasonably good handling qualities.

Applying eigenstructure assignment results in the following control gain matrix:

$$K = \begin{bmatrix} -0.1668 & -0.0027 & -0.1211 & -0.0683 \\ 3.5802 & -0.0015 & 0.0086 & -0.2024 \end{bmatrix}$$

This gain matrix gives closed-loop eigenvalues, which are identical to those desired, and eigenvectors, which are close to the desired. Because the p and ϕ gains were small in both channels and the r gain was small in the roll command channel, they were eliminated, resulting in a simplified control:

$$K_s = \begin{bmatrix} -0.1668 & 0 & 0 & 0 \\ 3.5802 & 0 & 0 & -0.2024 \end{bmatrix}$$

The closed-loop eigenvalues for this controller are as follows: Dutch roll $-0.5149 \pm 1.9723i$ ($\omega = 2.0384$, $\zeta = 0.2526$), roll subsidence −4.5401, and spiral −0.0331, and the eigenvectors are

$$\begin{matrix} 0.2133 \pm 0.0332i, & 0.0012, & -0.0122 \\ 0.2319 \pm 0.3168i, & 0.2406, & -0.9977 \\ 0.7395 \pm 0.2926i, & -0.9706, & 0.0450 \\ -0.1301 \pm 0.3871i, & 0.0022, & -0.0493 \end{matrix}$$

The closed-loop eigenstructure is fairly close to the desired.

The elements of the desired and actual transfer function matrices between the inputs p_c and β_c and the outputs p and β are shown in Fig. 1. It can be seen that there is very little coupling between the side-slip and roll-rate command, i.e., a roll-rate command would result in an aileron roll. As specified, the roll rate to roll-rate command transfer function is first order with a rolloff frequency of 4 rad/s and the side slip to side-slip command transfer function is second order

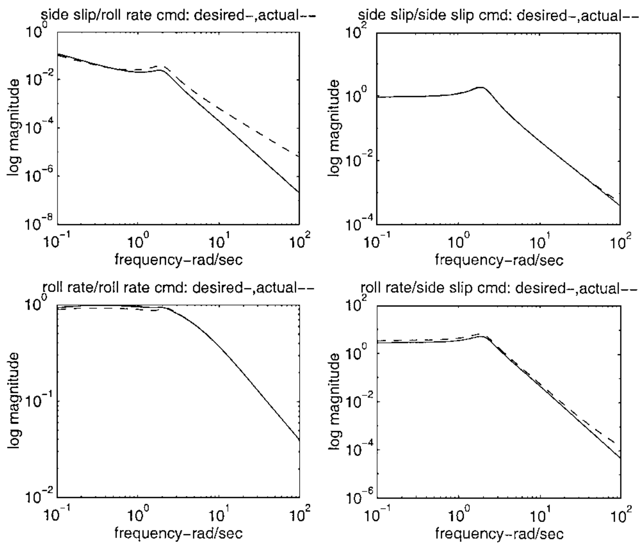

Fig. 1 Closed-loop transfer functions.

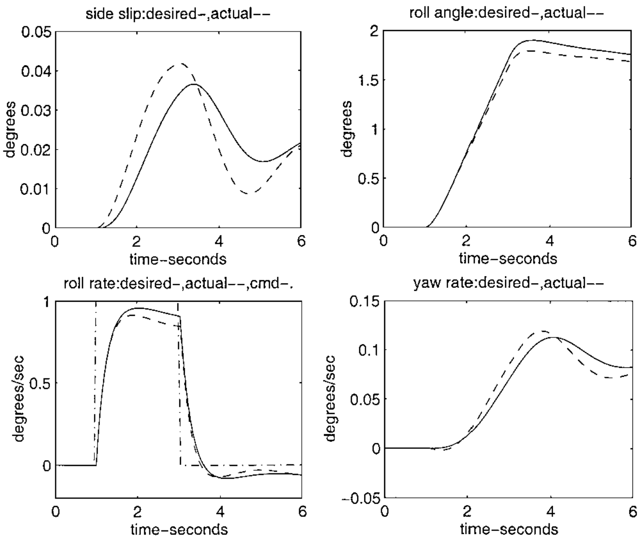

Fig. 2 Time history responses: roll-rate command.

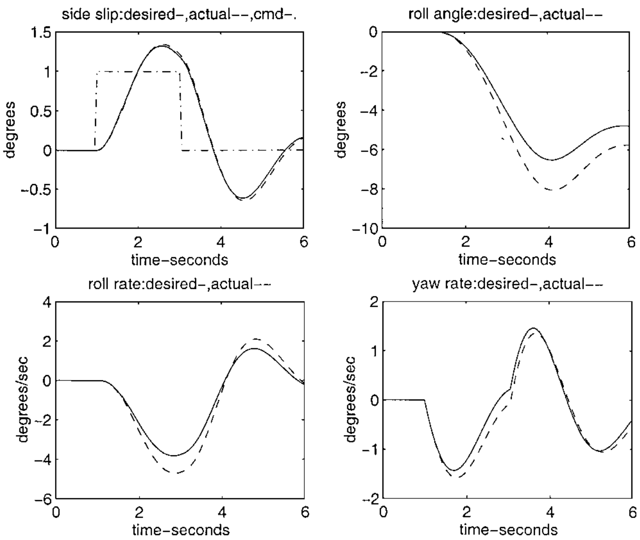

Fig. 3 Time history responses: side slip command.

with a natural frequency of 2 rad/s. There is coupling between side slip and roll rate as specified in the model for desired response.

Time history responses to a 2-s step, roll-rate command are shown in Fig. 2. It can be seen that the desired and actual responses provide first-order tracking of the command and that the directional response is very small. Figure 3 shows time history responses to a 2-s step, side-slip command. A second-order response is achieved and a positive side slip results in a negative roll response as is required for static lateral stability.

Because the aircraft is unstable, robustness of the controller is important. It was found that the size of the B matrix could be reduced by 18% before the aircraft became unstable.

Conclusions

The method outlined in this Note provides a simple model for generating desired eigenvalues and eigenvectors for use in eigenstructure design of lateral-directional controllers for aircraft.

Acknowledgment

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References

[1]Andry, A. N., Shapiro, E. Y., and Chung, J. C., "Eigenstructure Assignment for Linear Systems," *IEEE Transactions on Aerospace Electronic Systems*, Vol. AES-19, Sept. 1983, pp. 711–729.

[2]Sobel, K. M., and Shapiro, E. Y., "Application of Eigenspace Assignment to Lateral Translation and Yaw Pointing Flight Control," *Proceedings of the 23rd IEEE Conference on Decision and Control*, Inst. of Electrical and Electronics Engineers, New York, 1984, pp. 1423–1438.

[3]Apkarian, P., "Structured Stability Robustness Improvement by Eigenspace Techniques: A Hybrid Methodology," *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 2, 1989, pp. 162–168.

[4]Moore, B. C., "On the Flexibility Offered by Full State Feedback in Multivariable Systems Beyond Closed Loop Eigenvalue Assignment," *IEEE Transactions on Automatic Control*, Vol. 21, Oct. 1976, pp. 682–691.

[5]Garrard, W. L., Low, E., and Prouty, S., "Design of Attitude and Rate Command Systems for Helicopters Using Eigenstructure Assignment," *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 6, 1989, pp. 783–792.

[6]Low, E., and Garrard, W. L., "Design of Flight Control Systems to Meet Rotorcraft Handling Qualities Specifications," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 1, 1993, pp. 69–78.

[7]Mengali, G., "Mixed-Linear Quadratic/Eigenstructure Assignment Strategy for Design of Stability Augmentation Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 6, 1996, pp. 1231–1238.

[8]"Military Standard Flying Qualities of Piloted Aircraft," MIL-STD-11797A, Dept. of Defense, Washington, DC, Jan. 1990.

[9]McReur, D., Ashkenas, I., and Graham, D., *Aircraft Dynamics and Automatic Control*, Princeton Univ. Press, Princeton, NJ, 1973, Chap. 6.

[10]Paul, R., and Garrard, W., "Dynamics and Control of Tailless Aircraft," AIAA Paper 97-3776, Aug. 1997.

Transfer Function Parameter Changes Due to Structural Damage

Jiann-Shiun Lew*
Tennessee State University,
Nashville, Tennessee 37203-3401

Introduction

THE damage detection of large flexible structures is an important and challenging task in the system identification and structural dynamics communities. Recently, various damage detection approaches have been proposed.[1–3] Most of them use the finite element model (FEM) update techniques. In general, the FEM update techniques require many sensors to measure mode shapes, but the number of sensors is limited in practical applications. Recently, a correlation approach[1] was developed for damage detection based on the comparison of the transfer function parameter change of the tested system and the change due to damage. Only a few sensors are required for this approach. The results of a comparison of this method with some other methods show that this method is more robust to model inaccuracy such as the error due to noise than the other methods.[4]

This Note presents a novel study of the transfer function parameter change due to structural damage by applying this correlation approach. This extends the correlation approach to the study of the characteristics of the parameter change due to structural damage such as the reduction of element stiffness. The characteristics of the parameter change play an important role in pattern recognition for methods such as neural network techniques, which can be applied to structural damage detection. Because only a few sensors are required for the correlation approach, the placement of sensors is important. This Note also addresses the issue of optimal sensor placement for structural damage detection. The presented sensor placement technique is based on the sensitivity of the transfer function parameter to structural damage.

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*Research Professor, Center of Excellence in Information Systems. Member AIAA.